Minimum χ^2 Method

By definition of Alternate method of MLE

$$\chi^2 = \sum_{j=0}^{n} \left(n_j - np_j(\theta)\right)^2 \Big/ np_j(\theta)$$

(A)

Where $np_j(\theta)$ is the expected frequencies and n=n**0**+n1+n2+…………………+nk are the observed frequencies.

Calculate the expected cell frequencies (probabilities) and put in eq (A).

Then minimize i.e χ^2 =0 and get the estimates of the parameter' θ ' which is known as minimum χ^2 –estimate and the producer leading towards this estimate is called minimum χ^2 - method .

i.e. χ^2 method is the alternative method of MLE when its application is difficult or simplification become tough then we use minimum - χ^2 method instead of MLE.

Question # 1

Let $x_1, x_2, x_3 \ldots\ldots\ldots\ldots x_n$ be a random sample from the Bernoulli distribution

$$f(x;\theta) = \theta^x (1-\theta)^{1-x}$$

Then find the minimum χ^2 estimate of the parameter θ.

Solution:

$$f(x,0) = \theta^x (1-\theta)^{1-x}$$

$$p_j(\theta) = \theta^j (1-\theta)^{1-j}$$

At

$$j = 0$$

$$p_0(\theta) = \theta(1-\theta)^{1-0} = (1-\theta)$$

$$j = 1$$

$$p_1(\theta) = \theta(1-\theta)^{1-1}$$

$$= \theta$$

By Definition of Minimum χ^2 -Method

$$\chi^2 = \sum_{j=0}^{k} \left[n_j - np_j(\theta)\right]^2 \Big/ np_j(\theta)$$

$$= \sum_{j=0}^{1} \left[n_j - np_j(\theta)\right]^2 \Big/ np_j(\theta)$$

$$= \frac{\left[n_0 - np_0(\theta)\right]^2}{np_0(\theta)} + \frac{\left[n_1 - np_1(\theta)\right]^2}{np_1(\theta)}$$

$$= \frac{\left[n_0 - n(1-\theta)\right]^2}{n(1-\theta)} + \frac{\left[n_1 - n(\theta)\right]^2}{n(\theta)}$$

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$$= \frac{\left[n_0 - n(1-\theta)\right]^2}{n(1-\theta)} + \frac{\left[n_1 - n(\theta)\right]^2}{n(\theta)}$$

$$\sum_{j=1}^{k} n_j = n$$

$n_0 + n_1 = n$ $\qquad$ $n_0 = n - n_1$

$$\chi^2 = \frac{\left[n - n_1 - n + n\theta\right]^2}{n(1-\theta)} + \frac{(n_1 - n\theta)^2}{n\theta}$$

$$= \frac{(n\theta - n_1)^2}{n(1-\theta)} + \frac{(n_1 - n\theta)^2}{n\theta}$$

$$= \frac{1}{n}\left[\frac{\theta\left(n^2\theta^2 + n_1^2 - 2n_1 n\theta\right) + (1-\theta)(n_1^2 + n^2\theta^2 - 2n_1 n\theta}{\theta(1-\theta)}\right]$$

As

$$\chi^2 = o$$

$$0 = \frac{\theta\left(n^2\theta^2 + n_1^2 - 2n_1 n\theta\right) + \left(n_1^2 + n^2\theta^2 - 2n_1 n\theta\right) - \theta\left(n^2\theta^2 + n_1^2 - 2n_1 n\theta\right)}{\theta(1-\theta)}$$

$$o = n_1^2 + n^2\theta^2 - 2n_1 n\theta$$

$$0 = (n_1 - n\theta)^2$$

Taking square root on both sides

$$0 = (n_1 - n\theta)$$

$$n_1 = n\theta$$

$$\theta = \frac{n_1}{n}$$